An Investigation into the Magnetic Behaviour of Iron and some other Metals under the Oscillatory Discharge from a Condenser.

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Introduction.

1. The principal object of this investigation was the determination of the relation between magnetising force and permeability when iron or nickel is subjected to alternating magnetisation obtained from the oscillatory discharge of a condenser, especially with a view to finding the permeability with large magnetising forces.

A good deal of work has been done on this subject, that which is most akin to that described in this paper has been carried out by Battelli and Magri,* who have observed the phenomenon of the oscillatory discharge with varying period as first described by the author in a letter to 'Nature.'† The highest value of magnetising force employed in their experiments was about 700 C.G.S. units whereas the range covered in the tests herein described was from 500 to 5000 C.G.S. units.

Other workers in the same field have been Hertz,‡ Jouast,§ Schames,|| Zahn,¶ Varley,** Hemsalech,†† Zenneck,‡‡ Fleming,§§ Birkeland,|||| Sir J. J. Thomson,¶¶ C. E. St. John,*** Klemincic,††† Trowbridge,‡‡‡ and Pellat.§§§

* 'Accad. Lincei Atti,' vol. 15, pp. 63–74.
† 'Nature,' vol. 62, p. 413.
‡ 'Wied. Ann.,' vol. 26, p. 427.
§ 'Soc. Int. Elect. Bull.,' 1911, Ser. 3, vol. 1, pp. 49–57.
|| 'Ann. d. Physik,' vol. 27 (1), pp. 64–82.
¶ 'Deutsch. Phys. Gesell. Verh.,' vol. 11 (8), pp. 185–204; vol. 11 (18), pp. 377–381.
** 'Phil. Mag.' (3), vol. 3, pp. 500–512.
†† 'Comptes Rendus,' vol. 132, pp. 917–920.
‡‡ 'Ann. d. Physik,' vol. 12 (4), pp. 869–874.
§§ 'Roy. Soc. Proc.,' vol. 70, p. 398.
|||| 'Comptes Rendus,' June, 1894.
¶¶ 'Recent Researches in Electricity and Magnetism,' p. 302.
*** 'Phil. Mag.' (5), vol. 38, p. 425.
††† 'Wien. Akad. Sitz.,' vol. 107, p. 330.
‡‡‡ 'Ann. d. Physik,' vol. 12 (4), pp. 869–874.
§§§ 'Comptes Rendus,' June 11, 1894.

Method of Observation.

2. The method adopted in this investigation was to photograph the image of the spark produced by the condenser discharge reflected from a revolving mirror (see fig. 1). This method was similar to that originally adopted by

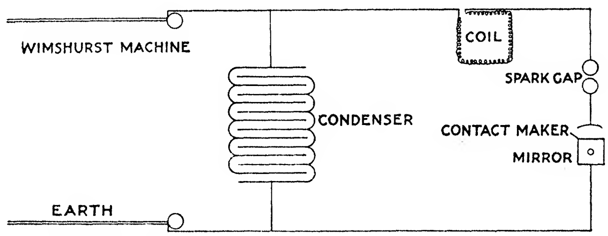

Fig. 1.—Diagram to show arrangement of circuit.

Feddersen and Schiller, but in this case the surface of the mirror was flat and the light from the spark was focussed by a lens on to the photographic plate.

The mirror used in most of the experiments was formed with four silvered glass plates supported in a square aluminium frame, each mirror being 1 inch by 4 inches. This could be run with safety up to 100 revolutions per second. For higher speeds a small silver mirror was used with a reflecting face $\frac{1}{2}$ inch by 2 inches, which was run up to 230 revolutions per second. The speed of the mirror was determined by a contact wheel driven by a worm cut in the shaft of the mirror, the contacts being arranged in one of the circuits of a double chronograph, the second circuit was connected to a clock giving seconds, and the speed could be measured to within $\frac{1}{4}$ per cent. The spark gap was made with brass balls and its length was measured by a micrometer. The circuit of the spark was closed through a contact-maker formed of a brush of fine wires attached to the frame of the mirror, which rubbed against a brass plate. This contact-maker gave excellent results, chiefly because of the high speed at which the brush was revolved.

Condensers.

3. The condensers used consisted, in the first experiments, of a number of Leyden jars each having approximately 0·004 microfarad capacity. A second small condenser, sometimes used, consisted of nine plates of glass, 2 feet square, on which were pasted sheets of tinfoil 18 inches square, the glass being carefully shellacked and the whole well insulated.

The third condenser (the one chiefly used) consisted of 10 sets of 10 glass plates 3 feet square, coated with tinfoil 2 feet square, the glass being

approximately 0·063 inch thick. The capacity of each set of 10 plates was about 0·14 microfarad, giving a total capacity of nearly 1·4 microfarads, which would stand, if necessary, 30,000 volts. The condenser, however, was never used at so high a voltage, except for test purposes; the normal discharge, as stated above, taking place at about 10,000 volts.

An air condenser was also used consisting of 30 sheets of zinc 18 inches square, placed 0·25 inch distant from each other, and having a capacity = 0·010 microfarad. This latter condenser was not used in the special experiments under consideration, but merely to obtain a result to test how nearly the observed and calculated frequencies agreed for the coils used in the experiments. The capacity of these condensers was compared by a ballistic method with the capacity of a standard Elliott ½-microfarad condenser, which had been calibrated at Glasgow University by comparison with the standards there, and also by testing with the instruments at hand in the laboratory.

Coils.

4. The coil chiefly used was square in shape, made of a single layer of No. 16 guttapercha-covered wire wound on a tube of shellacked paper. The tube was made square, in order to facilitate the introduction of cores of suitable materials. And, by using a single-layer coil, it was possible to get a more accurate approximation to the apparent permeability of the substances introduced than with a coil of several layers, in which the proportion of the area of each layer influenced by the introduction of the coil varied for the

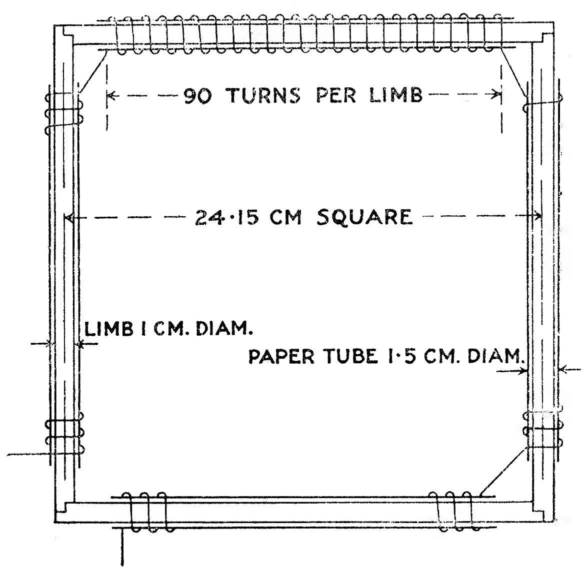

Fig. 2.

different layers. Each side of the square coil contained 90 turns, the diameter of each turn being approximately 1·5 cm. (see fig. 2).

The object of using a square coil was to obtain, as far as possible, a closed magnetic circuit of the substances which were introduced, generally iron wire cores, as by this means calculation would be simplified.

With the assistance of Mr. Duddell, to whom the author is much indebted, a very sensitive measuring apparatus was arranged, with the permission of the late Prof. Ayrton, at the Central Technical College, for the measurement of this self-induction.

With this apparatus the value $L = 4{\cdot}1 \times 10^{-5}$ henrys was obtained correct to ± 1 per cent. for the single-layer coil. The resistance of this coil at 14° C. was 0·63 ohm. Another square coil was used to obtain lower frequencies; in this there were seven layers of wire on each side of the square, the whole containing 2670 turns of wire. The self-induction of this coil was also carefully measured with a secohmmeter, and found to be $5{\cdot}031 \pm 0{\cdot}012$ millihenrys. The resistance of the coil at 14·3° C. was 8·18 ohms. A third coil, used chiefly for preliminary experiments, consisted of a single bobbin of the same wire wound in seven layers; this contained 750 turns of wire, and had a self-induction of approximately 1 millihenry. Its self-induction was not measured by the secohmmeter, but was obtained by comparison with the square seven-layer coil by means of the air condenser. The results of these experiments are given later.

Measurement of the Plates.

5. In order to obtain accurate measurement of the spark photographs, a large measuring micrometer with a low-power microscope was used, with a cross wire in the eye-piece. In most cases it was found possible to obtain measurements of not very quickly damped sparks accurately to within 2 per cent., but with sparks whose period varied the accuracy was very much lower, as the black spots were often spread out and asymmetrical.

Theory of Experiments.

6. From the fundamental equations for the discharge of a condenser of C farads capacity through a coil of self-induction L henrys, and with a total resistance of R ohms in the circuit, the frequency of oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{CL} - \frac{R^2}{4L^2}\right)}.$$

The current at any instant $= \dfrac{Q_0}{pLC} e^{-Rt/2L} \sin pt$ ampères,

where $\qquad p = 2\pi \times \text{frequency} \quad \text{and} \quad Q_0 = V_0 C,$

where V_0 is the P.D. on the spark gap just before discharge, measured in volts.

When the self-induction of the coil is variable, a graphical method of determining the frequency of oscillation has been used.*

Equation for the Induction in an Iron Core.

7. It will now be necessary to consider the case of a core of soft iron wires, which is magnetised by the rapidly alternating current flowing through the coil surrounding them. It may be assumed for simplicity that the cores of the square coil form a closed magnetic circuit.

In order to gain some idea of the way in which the magnetism dies away as it penetrates into the iron wire, the analysis for a single straight circular soft iron wire given by Sir J. J. Thomson† has been adopted. In a very fine wire, such as is used in the experiments under consideration, the cross magnetisation may be neglected.

The total flux (c) through the wire

$$= \left(\int_0^a A J_0(jnr)\, 2\pi r dr\right) e^{jpt}.$$

But $$\int r J_0(jnr)\, dr = (jr/n)\, J_0'(jnr),$$

from the known properties of Bessel functions.

Since $J_0(0) = 0$, $$C = \frac{2\pi ja}{n} J_0'(jna)\, A e^{jpt},$$

or, substituting for A, $$C = \frac{2\pi\mu H ja J_0'(jna)}{n J_0(jna)} e^{jpt}.$$

In calculating the value of C from this expression, the value $n^2 = 4\pi\mu jp/\sigma$ has been used, leakage being neglected.

Damping of Oscillations.

8. This is an important consideration, as the most marked effect produced by solid iron cores (or, in fact, solid cores of any metal) is the greatly increased damping of the oscillations; while even with fine iron wire cores the number of vibrations in many cases is reduced from 10 or 15 to 3 or 4.

According to the investigation given above the damping factor is $E^{-Rt/2L}$; and accordingly, as L increases, R remaining constant (the change caused by introducing the iron wire cores), $R/2L$ is diminished, and hence the damping should be less rapid with the larger self-induction. It is clear, therefore, that

* 'Phil. Mag.,' January, 1903, p. 155.

† 'Recent Researches in Magnetism and Electricity,' p. 304.

the cause of the very rapid damping must be the absorption of energy by the cores introduced, either by hysteresis or by eddy currents, or both.

Using the same notation as above, in which the magnetic force is represented by the real part of H, the magnetic induction being assumed uniform along the axis of the wire, the rate of heat production per unit length of wire due to eddy currents is equal to the mean value of—

$$\tfrac{1}{2}a\left(\text{real part of } \frac{p}{n}\frac{J_0'(jna)}{J_0(jna)}\mu H e^{jpt}\right) H \cos pt,$$

where μ = magnetic permeability,
$p = 2\pi \times$ frequency,
σ = specific resistance of iron,
a = radius of iron wire.

When na is small the heat produced $= N/16\sigma\pi\mu^2p^2a^4H^2$* per unit length of core per second, N being the number of wires and H the maximum magnetising force.

The experiments of Sir J. J. Thomson (*loc. cit.*) appeared to show that the hysteresis of iron accounted for a considerable part of the absorption of energy.

In these experiments it will be shown that hysteresis loss is almost negligible as compared with eddy-current loss.

Results of Experiments.

Part I.

9. The first series of experiments† was made in order to find how nearly the frequency determined by experiment agreed with that calculated from the observed self-induction and capacities used in the tests.

The tests made with the air condenser may first be considered. The capacity of the air condenser was obtained both by calculation and by comparison with the standard ½-microfarad condenser. The capacity found in this way = 0·0102 microfarads. On account of the bending of the plates, it was found impossible to obtain a spark more than 0·6 mm. long, and some difficulty was found in photographing this, as well as in maintaining the discharge by the large Wimshurst machine.

A series of determinations was first made of the self-induction of the circuit apart from the coil. Using the small coil, the time for a complete

* 'Recent Researches in Electricity and Magnetism,' J. J. Thomson, p. 320.

† These results are not given at great length as the paper by Battelli and Magri ('Phil. Mag.,' June, 1903, p. 620) covers the subject quite fully.

oscillation of the circuit with the coil in position = T_c (see Table). The time for a complete oscillation without the coil = T_e.

	T_c (with coil).	T_e (without coil).	Ratio.
With Sets II–V of glass condenser	3·25	1·18	2·75
	3·25	1·16	2·80
Sets II–VIII of glass condenser	4·46	1·56	2·86

The mean value of T_c/T_e = 2·81.

Hence, calculating the self-induction of the circuit, $L_e = 0{\cdot}6 \times 10^{-5}$ henry.

A typical result with the air condenser and large coil is given below:—

Self-induction = 5·03 millihenrys, which, with the self-induction of the circuit, gives a total self-induction = 5·036 millihenrys.

Capacity	0·0102 microfarad,
Theoretical time for a complete vibration......	$4{\cdot}50 \times 10^{-5}$ sec.,
Length of half-oscillation on the plate	0·066 inch,
Time for 100 revolutions of the mirror.........	2·89 secs.,
Time for complete vibration	$4{\cdot}45 \times 10^{-5}$ sec.

The agreement is within the limits of experimental error.

This measurement was repeated, in order to verify the result:—

Length of half-oscillation.................	0·0114 (mean of six readings),
Time for 100 revolutions of the mirror	1·68 secs.,
Time for complete vibration	$4{\cdot}48 \times 10^{-5}$ sec.,

giving a still closer agreement.

An attempt was made to use the single-layer square coil, but it was not found possible to obtain a spark photograph which could be measured with any approach to accuracy. The results have not, therefore, been given.

10. A series of tests was made with the large glass condensers and the small coil, and the time of oscillation for each set of condensers carefully measured. These results are tabulated below:—

Number of sets of condensers.	K_c in microfarads.	$T \times 10^{-5}$.
I	0·137	1·58
I–II	0·279	2·28
I–III	0·415	2·76
I–IV	0·554	3·18
I–V	0·690	3·55
I–VI	0·82	3·84
I–VII	0·95	4·2
I–VIII..............	1·08	4·46
I–IX	1·22	4·68
I–X	1·35	5·06

In all these experiments the spark length was 2·5 mm. and the values of T are accurate to within 2 per cent.

11. In order to see whether the resistance of the circuit can have any effect on the frequency of the oscillations, the effective resistance (R_1) for the

wire has been calculated from the expressions originally given by Lord Rayleigh. For the highest frequencies used—

$$R_1 = 1\cdot18 \times (\text{resistance to steady current}).$$

Since $R = 0\cdot38$ ohm, $R_1 = 0\cdot45$ ohm.

For this frequency,

$$1/LC = 2\cdot6 \times 10^{11} \quad \text{and} \quad (R/2L)^2 = 0\cdot014 \times 10^{11}.$$

The effect of the resistance of the circuit in this case is therefore negligible.

For the lower frequencies, when using the single-layer square coil, the total resistance neglecting the spark may be taken = 1 ohm,

$$\frac{1}{LC} = \frac{10^{11}}{5\cdot7} \quad \text{and} \quad \left(\frac{R}{2L}\right)^2 = \frac{10^{11}}{670},$$

so that in this case also the correction is less than 1 per cent. When the coil is used with an iron core, the value of L is increased, and the correction will therefore be smaller still.

It will be shown later that when the maximum current during an oscillation of the spark falls to less than 200 ampères, the resistance of the spark increases to about 2 ohms, and may be expected to become larger still as the current through the spark diminishes still further. Under certain conditions, therefore, the spark resistance will have to be taken into account.

Part II.

12. The results of the experiments made to determine the effect of currents at high frequencies on iron may now be considered. With frequencies varying from 100,000 to 5000 per second, the effect of an iron or nickel wire core is to produce a spark discharge which has no constant frequency, but in which the period for a half-oscillation rapidly increases as the discharge becomes damped by energy absorption.* Having observed the effect, the explanation is obvious. It is clear that at these frequencies iron retains very much the same properties as it does under a steady magnetic field, *i.e.* the permeability of the iron increases as the flux density in the iron diminishes.

13. Experiments have been made with a variety of soft iron wire cores, and it will be convenient first to consider the effects produced with discharges from the set of ten large glass condensers through the single-layer coil, when the latter is arranged with an iron wire core.

* This phenomenon has now been observed by Battelli and Magri (*loc. cit.*).

The values of K_c in the above table have been used in the calculations, and the results are tabulated below.

t = time in seconds for 100 complete revolutions of mirror,

l_1 = length of quarter-oscillation from pilot spark at the beginning of the discharge to the first maximum current or blob on the plate,

l_2 = distance between first and second maxima,

l_3 = distance between second and third maxima,

l_4 = distance between third and fourth maxima, and so on.

The distances between the maximum values have been taken, as it was impossible to determine with any approach to accuracy the distance between successive minima.* To obtain the value of l_1', l_2', l_3', etc., which represent the lengths between successive zero values of current, the following method has been adopted. Taking first l_1 and l_2, (l_1+l_2) has been assumed to form the sum of three terms of an arithmetical progression, the first term of which is l_1, l_1' represents the sum of the first two terms.

The values of L_1, L_2, etc., are calculated from these figures.

The method of drawing the discharge curve already referred to,† when the B-H curve for the iron is known, has been applied in certain cases, and the result of one of these calculations is given in fig. 3.

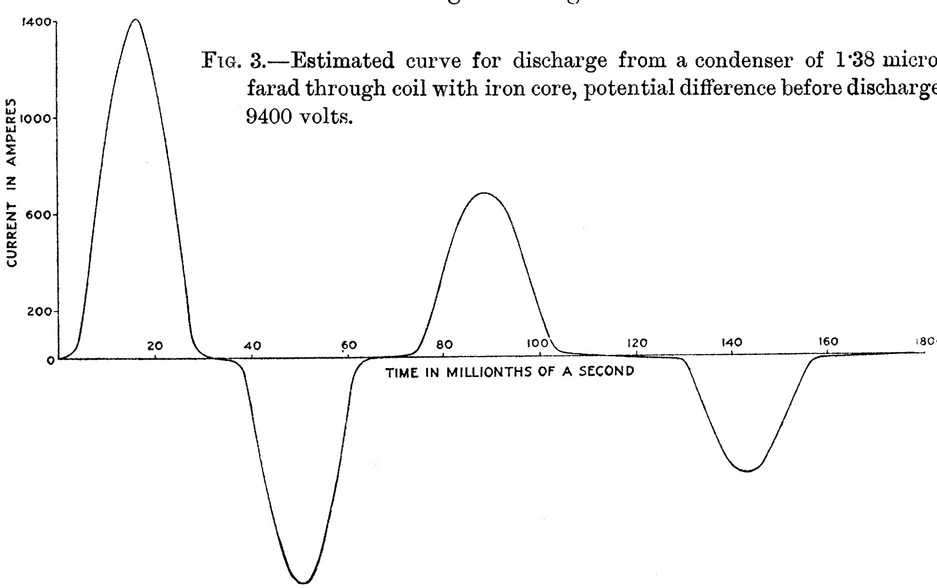

FIG. 3.—Estimated curve for discharge from a condenser of 1·38 microfarad through coil with iron core, potential difference before discharge, 9400 volts.

To obtain the "effective permeability" of the iron the lines of force may be assumed to be of uniform density; on the introduction of an iron

* The reason for this is clear from the curves drawn graphically in fig. 5.

† 'Phil. Mag.,' January, 1903, p. 155.

core a certain number of lines go through the iron. We may put, therefore, $L_0 = KA_0$, where A_0 is the area of one turn of the coil (calculated from the mean value of the self-induction) and K is a constant.

When a core is introduced $L = K[(A_0 - A_1) + \mu_e A_1]$, where A_1 is the area of the iron core, and μ_e is a quantity which may be designated the "effective" permeability.

Hence $$\mu_e = 1 + \frac{(L - L_0) A_0}{A_1 L_0}.$$

The quantity μ_e may be defined as the permeability of the iron which would give the same period of oscillation for the spark as is obtained experimentally, the "effective permeability" being constant and independent of the value of the magnetising force.

With the single-layer square coil $A_0 = 2{\cdot}29$ sq. cm.

						sq. cm.	
(1) Area of section of the No.		28	S.W.G.	iron	wire core	= 0·64.	$\frac{A_0}{A_1} = 3{\cdot}55.$
(2)	,,	18	,,		,,	= 0·70.	$\frac{A_0}{A_1} = 3{\cdot}06.$
(3)	,,	24	,,		,,	= 0·61.	$\frac{A_0}{A_1} = 3{\cdot}7.$
(4)	,,	28	,,	nickel	,,	= 0·58.	$\frac{A_0}{A_1} = 3{\cdot}9.$

14. In the following table are shown the results obtained when the discharge from No. 1 set of the large condenser flows through the single-layer square coil into which a core [(1) above] made up of 550 No. 28 S.W.G. iron wires has been inserted, the spark gap is 2·5 mm., corresponding to a P.D. before discharge of 9400 volts.

Large condenser, No. I set.—$T_0 = 1{\cdot}58 \times 10^{-5}$ sec., $L_0 = 4{\cdot}1 \times 10^{-5}$, $l = 0{\cdot}6 \times 10^{-5}$, volts = 9400, $t = 2{\cdot}26$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·03	0·068	$3{\cdot}6 \times 10^{-5}$	24·1	18·4
0·085	0·118	$6{\cdot}25 \times 10^{-5}$	73·0	60·0
0·17	($\frac{1}{4}$)0·098	$10{\cdot}4 \times 10^{-5}$	195·0	164·0

15. In the first place, the value of the maximum current during the first half-oscillation was estimated, and was found to be 210 ampères.

The corresponding value of the magnetising force due to this current = 1000 C.G.S. units approximately.

The value of B corresponding to this, obtained from Ewing's results for Low Moor wrought iron = 22×10^3, giving a value of $\mu = 22$ approximately, a result which is of the same order of magnitude as the effective permeability calculated above.

16. By the calculation made below, it will be seen that allowance for "skin" effect makes very little difference in the value of μ. Taking

$$\mu = 19, \qquad \sigma = 1{\cdot}2 \times 10^5, \qquad p = 1{\cdot}75 \times 10^5,$$

from above, and making the necessary substitutions, the magnetic induction through a single wire

$$C = 0{\cdot}022\,H \cos \overline{pt - \phi}, \quad \text{where} \quad \tan\phi = 0{\cdot}134,$$

giving an effective permeability which does not differ appreciably from 19.

This result shows, however, that there is a lag in the maximum induction through a wire, behind the magnetising force of about 9° on the assumption that the permeability of the iron is constant.

17. It will now be necessary to take into account the loss of energy during a half-oscillation of the spark.

The losses of energy may be classified under four heads :—

(1) Loss (of heat, light, and sound) in the spark itself.

(2) Loss of energy in the wires of the circuit due to the passage of the current (C^2R loss). (This is allowed for in the damping factor.)

(3) Loss of energy in the cores due to hysteresis of the iron.

(4) Loss of energy in the cores due to eddy currents.

The losses due to the spark, and to Joulian heat loss in the wires, occur in exactly the same way in discharges through coils having no cores (in which case there are usually 10 or more half-oscillations on the negative) as they do when iron cores are introduced, in which there are usually not more than two or three half-oscillations visible, with the smaller condensers. The main loss of energy for these discharges must therefore be sought in other directions. (With the discharges from the largest condenser a greater number of oscillations are visible with the iron cores in position, but for these discharges the ohmic loss is much more considerable.)

18. Considering first the losses due to hysteresis. The iron used for the core was very soft and well annealed, and, as a first approximation, a loss of 20,000 ergs per cubic centimetre for a complete cycle has been assumed, or 10,000 ergs for a half cycle. In making this assumption, the very high flux in the iron has to be taken into account, coupled with the statement of M. Klemencic* and others that the hysteresis loss is greater at high

* 'Wien. Akad. Sitzberg,' 1898, vol. 107, pp. 330–360. M. Klemencic seems to have determined the hysteresis loss by a determination of the decrement of current in a Leyden jar discharge. No allusion is made to corrections which have to be introduced for eddy current losses. See also Warburg and Hönig, 'Wied. Ann.,' vol. 20, p. 814; 'Phil. Mag.,' September, 1889; Battelli and Magri, 'Accad. Lincei Atti,' vol. 15, pp. 485–492; Corbino, 'Accad. Lincei Atti,' vol. 16, pp. 167–170; 'Atti dell' Assoc. Elett.,' vol. 7, p. 606; F. Piola, 'Elett. Romi,' vol. 5, pp. 4–6.

frequencies than low. Now, the total volume of the iron in the cores = 62 c.c. approx. Hence the total loss of energy due to hysteresis for the half-oscillation = $0 \cdot 62 \times 10^6$ ergs. This loss is quite small as compared with the eddy-current losses.

The loss due to eddy currents during the first half-oscillation calculated from the formulæ given above = $23 \cdot 8 \times 10^6$ ergs. Adding to this the hysteresis loss as calculated above, the total iron loss = $2 \cdot 44 \times 10^7$ ergs. The ohmic loss due to the resistance of the coil is comparatively small. In this case, assuming the root mean square value of the current to be 0·707 of the maximum value, it amounts to $0 \cdot 4 \times 10^7$ ergs. for the first half-oscillation.

The total energy stored = $5 \cdot 95 \times 10^7$ ergs.

The value given above shows that the eddy-current and hysteresis losses amount to more than one-third of the total energy of the discharge during the first half-oscillation, whereas the loss due to ohmic resistance is only 0·07 of the total energy.

The damping of the oscillations with iron cores might be expected to be much more rapid than when the cores are not present, and this is precisely what is observed. The total number of half-oscillations on the plate with the iron cores is three, whereas without them ten half-oscillations, and even more, have been photographed.

This result confirms the observation that the permeability of the iron is not very different at this frequency from that observed under similar conditions with a steady magnetising force, for, if the magnetic induction were very different from the value that has been assumed, the total calculated eddy-current loss would not have agreed so nearly with experiment.

19. The other results obtained with the set of large condensers may now be dealt with. These will correspond with lower frequencies than those considered in the preceding paragraphs. The results are tabulated according to the method described in § 13.

Large condensers, Nos. I–II.—$T_0 = 2 \cdot 28 \times 10^{-5}$, volts = 9400, $L_0 = 4 \cdot 1 \times 10^{-5}$ h., $l = 0 \cdot 6 \times 10^{-5}$ h.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·029	0·067	$3 \cdot 9 \times 10^{-5}$	13·2	8·8
0·086	0·10	$5 \cdot 9 \times 10^{-5}$	31·0	24·5
0·1	0·167	$9 \cdot 7 \times 10^{-5}$	85·0	72·0
0·28	(¼)0·17	$20 \cdot 0 \times 10^{-5}$	362·0	310·0

Maximum current during first half-oscillation = 380 ampères.
Maximum magnetising force „ „ = 1830 C.G.S. units.

Large condensers, Nos. I–III.—Capacity = 0·385 mf., $T = 2{\cdot}76 \times 10^{-5}$, volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}4$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·041	0·092	$5{\cdot}2 \times 10^{-5}$	16·0	11·3
0·112	0·125	$7{\cdot}0 \times 10^{-5}$	30·1	23·0
0·13	0·164	$9{\cdot}2 \times 10^{-5}$	52·0	42·0
0·22	(¼)0·125	$14{\cdot}0 \times 10^{-5}$	120·0	100·0

Maximum current during first half-oscillation = 430 ampères.
Maximum magnetising force ,, ,, = 2070 C.G.S. units.

Large condenser, Nos. I–IV.—$T_0 = 3{\cdot}18 \times 10^{-5}$, volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$h., $l = 0{\cdot}6 \times 10^{-5}$h., $t = 3{\cdot}17$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·031	0·075	$5{\cdot}5 \times 10^{-5}$	13·5	9·5
0·101	0·114	$8{\cdot}4 \times 10^{-5}$	32·0	25·0
0·114	0·131	$9{\cdot}7 \times 10^{-5}$	43·0	35·0
0·164	(¼)0·091	$13{\cdot}4 \times 10^{-5}$	83·0	68·0

Maximum current during first half-oscillation = 530 ampères.
Maximum magnetising force ,, ,, = 2550 C.G.S. units.

Large condenser, Nos. I–VI.—$T_0 = 3{\cdot}84 \times 10^{-5}$, volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$h., $t = 3{\cdot}13$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·034	0·076	$5{\cdot}6 \times 10^{-5}$	9·4	5·6
0·09	0·099	$7{\cdot}2 \times 10^{-5}$	16·0	11·3
0·105	0·114	$8{\cdot}3 \times 10^{-5}$	21·5	16·0
0·126	0·148	$10{\cdot}8 \times 10^{-5}$	36·5	29·0
0·18	(¼)0·098	$14{\cdot}3 \times 10^{-5}$	64·0	54·0

Maximum current during first half-oscillation = 770 ampères.
Maximum magnetising force ,, ,, = 3700 C.G.S. units.

Large condenser, Nos. I–X.—$T_0 = 5{\cdot}06 \times 10^{-5}$, volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 3{\cdot}1$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.	Maximum magnetising force.	T for complete oscillation, estimated from curve in fig. 6.
0·04	0·089	$6{\cdot}45 \times 10^{-5}$	7·0	3·5	5250 gauss	$6{\cdot}4 \times 10^{-5}$
0·107	0·116	$7{\cdot}7 \times 10^{-5}$	10·1	6·3	3100 ,,	$7{\cdot}2 \times 10^{-5}$
0·115	0·131	$9{\cdot}5 \times 10^{-5}$	16·0	11·3	1900 ,,	$8{\cdot}4 \times 10^{-5}$
0·16	0·194	$14{\cdot}0 \times 10^{-5}$	25·4	19·4	950 ,,	$13{\cdot}4 \times 10^{-5}$
0·23	0·272	$19{\cdot}7 \times 10^{-5}$	70·0	58·0	(?) 420 ,,	
0·33	(¼)0·17	$24{\cdot}6 \times 10^{-5}$	110	93		

Estimated maximum magnetising force first half-oscillation = 5250 C.G.S. units.

Considering, in the first instance, the first half-oscillation of the spark after the discharge begins, it will be noticed that when iron cores are introduced the change in frequency produced by using different capacities is comparatively slight, varying from 28,800 ∼ per second with 0·137 microfarad to 18,800 ∼ per second with 1·35 microfarad. It will also be noticed that the decrease of effective permeability with increase of capacity is not quite uniform, but this is amply accounted for by possible errors in observation.

A curve has been plotted between the values of the effective permeability and maximum magnetising force, and is shown in fig. 4.

20. Considering next the half-oscillations of the spark other than the first

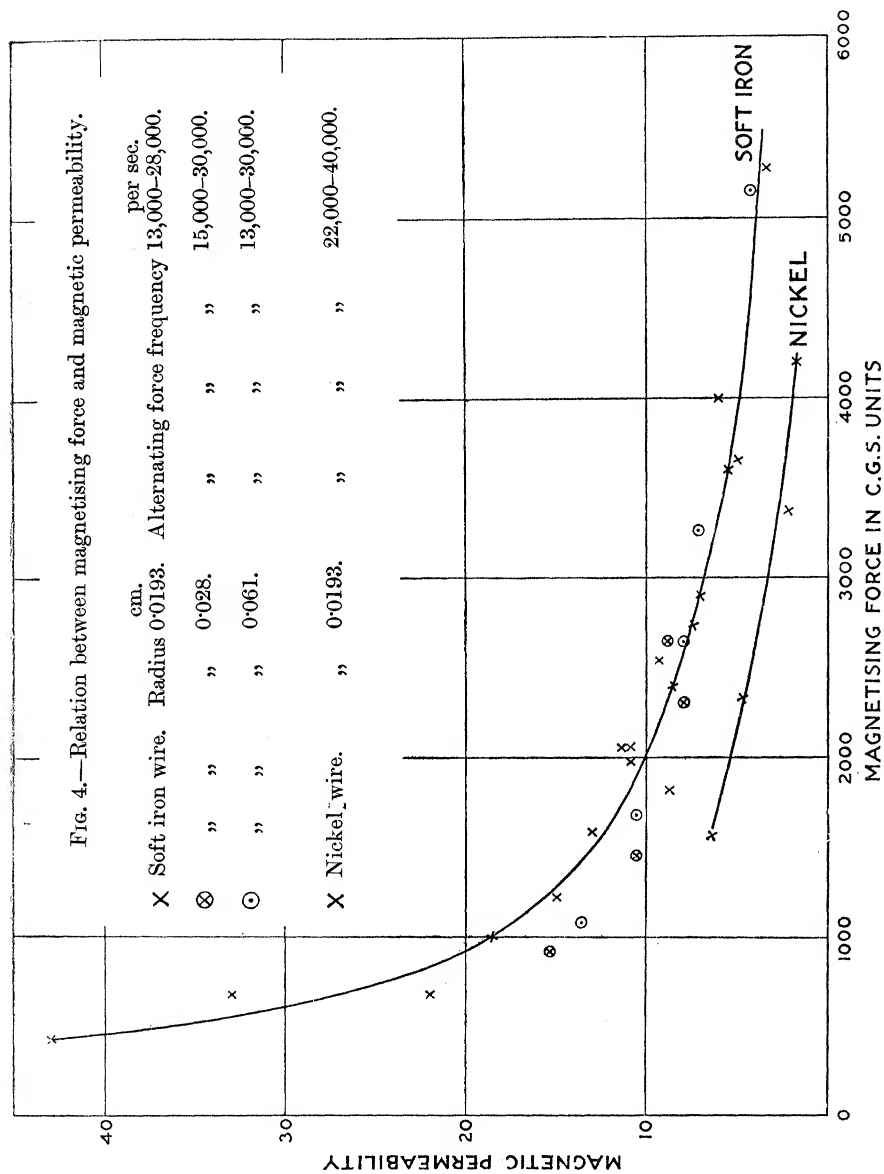

Fig. 4.—Relation between magnetising force and magnetic permeability.

it will be seen that the same law holds. Increase in capacity diminishes the apparent effective permeability. Thus the second half-oscillations with 0·137 microfarad corresponds with a greater value of the effective permeability.

It is noticeable from these figures that the damping of the discharge is

much less rapid with the larger capacities, the number of oscillations visible with the larger condenser being nearly as great with as without iron cores in the coil. This is due to the great increase in the relative importance of the ohmic losses with the larger condenser. Taking, for example, the discharge from the condenser with a capacity of 1·35 microfarads, the ohmic loss during the first half-oscillation is approximately 16×10^7 ergs, whereas the eddy-current loss in the core is less than that calculated in § 18, and is approximately 2×10^7 ergs.

The resistance loss in the circuit is therefore the predominant factor in determining the decrement of the spark, and it is only at the end of the spark that eddy-current loss becomes important. The great ohmic loss was also noticeable from the fact that after two or three discharges with the large condenser the coil became perceptibly warm. The discharge curve for this condenser has been drawn by the method already mentioned (see fig. 3),* the eddy-current loss being taken into account by suitably reducing the voltage at the beginning of each half-oscillation when drawing the curve. The times for a complete oscillation have been determined from the curve, and are tabulated with the results found experimentally. These figures agree within limits of experimental error with the spark photographs, and confirm the fact that the permeability of the iron at these frequencies is not very different from that under steady conditions. It will be noticed that the maximum current calculated on the assumption that sine waves are being dealt with is over 25 per cent. less than that estimated by the graphical analysis.

The figures calculated from the formula for the maximum magnetising current have been used in drawing the curves in fig. 3, and these are the values given in the tables. The root mean square or virtual current estimated from the graphical curves differs in most cases by not more than 6 per cent. from that obtained by assuming the "calculated" current to be of sine wave shape.

From the results obtained with the largest condenser an attempt has been made to determine the magnitude of the resistance of the spark itself.

From the values calculated from the "effective" permeability the corresponding value of "H" for each half-oscillation of the spark can be found, and hence the maximum value of the current, assuming a sine wave shape for the curve as a first approximation. From this value for the current, the maximum voltage at the condenser at the beginning of each half-oscillation has been determined, and hence the loss of energy during each half-oscillation

* The values of spark resistance referred to later have been taken account of in drawing this curve.

has been estimated. These results are tabulated below. The loss in energy during each half-oscillation has been assumed to be due to (1) ohmic resistance, (2) eddy current in the core, (3) hysteresis in the core. Dielectric hysteresis in the glass has not been taken into account. The total for these losses is also shown in the table below. If a spark resistance of 0·5 ohm is assumed, the losses are increased to the values shown in the last column, which agree quite closely with those calculated from the loss in voltage by the condenser. Although too much value should not be attached to the actual figure assumed for the resistance of the spark, it shows that with currents of this magnitude, the resistance is not large enough to cause any perceptible difference in the frequency of the oscillation.*

Large condenser, I–X.—Capacity 1·38 microfarad.

	Estimated value of H.	Estimated voltage at beginning of oscillation.	Loss in energy during half-oscillation.	Estimated total loss neglecting spark resistance.	Estimated total loss assuming spark resistance = 0·5 ohm.
1st half-oscillation	5250	9400	$33 \cdot 5 \times 10^7$ ergs.	$19 \cdot 3 \times 10^7$ ergs.	$33 \cdot 0 \times 10^7$ ergs.
2nd " "	3100	6300	$12 \cdot 4 \times 10^7$ "	$10 \cdot 2 \times 10^7$ "	$13 \cdot 6 \times 10^7$ "
3rd " "	1900	4680	$6 \cdot 9 \times 10^7$ "	$5 \cdot 26 \times 10^7$ "	$6 \cdot 9 \times 10^7$ "
4th " "	950	3400	$4 \cdot 9 \times 10^7$ "	$2 \cdot 50 \times 10^7$ "	$3 \cdot 0 \times 10^7$ "
5th " "	420 q.p.	2100			

N.B.—If the value of the spark resistance for the 4th half-oscillation be assumed equal to 2 ohms, the total energy loss during the 4th half-oscillation becomes $4 \cdot 9 \times 10^7$ ergs.

Similar calculations have been made on other typical discharges, in particular, with the discharge of a condenser of 1 microfarad at an initial potential difference of 7800 volts; the average resistance for the spark which it is necessary to assume to bring the energy losses into agreement is 0·48 ohm at 770 ampères, 0·76 ohm at 440 ampères, and 1·2 ohms at 230 ampères. The accuracy of these figures is not great, and they are of value as showing the order of magnitude of the quantity involved, and in this respect they confirm the results which have been given above.

21. Experiments were made in order to determine how the spark length, and, consequently, the potential difference at the spark gap just before the discharges, affected the observed effect. It is clear that a shorter spark will give a smaller maximum current during the first half-oscillation, and hence, according to the above results, the apparent self-induction of the coil should be increased. Below are given the results obtained with spark gaps 2 mm. and 1·5 mm. in length.

* This result is in accordance with that obtained by Battelli and Magri, 'Phil. Mag.,' June, 1903, vol. 5, pp. 620–643.

Using the notation of § 13—

Large condenser, No. I.—$T_0 = 1{\cdot}58 \times 10^{-5}$ sec., $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}64$ sec., volts before discharge = 7800.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·05	0·101	$3{\cdot}9 \times 10^{-5}$	28	22·4
0·104	($\frac{1}{4}$)0·004 (?)	$4{\cdot}1 \times 10^{-5}$	31	25·5

Maximum current during first half-oscillation = 140 ampères.
Maximum magnetising force " " = 670 C.G.S. units.

Large condensers, Nos. I–II.—$T = 2{\cdot}28 \times 10^{-5}$ sec., volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$, $l = 0{\cdot}6 \times 10^{-5}$, $t = 2{\cdot}26$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·04	0·09	$4{\cdot}8 \times 10^{-5}$	20·2 henry	15
0·12	0·15	$8{\cdot}0 \times 10^{-5}$	57·0 "	47
0·18	($\frac{1}{4}$)0·103	$10{\cdot}9 \times 10^{-5}$	117·0 "	99

Maximum current during first half-oscillation = 255 ampères.
Maximum magnetising force " " = 1220 C.G.S. units.

Large condensers, Nos. I–IV.—$T_0 = 3{\cdot}18 \times 10^{-5}$ sec., volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}92$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·062	0·131	$5{\cdot}9 \times 10^{-5}$	15·6	11
0·152	0·18	$8{\cdot}1 \times 10^{-5}$	30·0	23
0·22	($\frac{1}{4}$)0·12	$10{\cdot}8 \times 10^{-5}$	54·0	44

Maximum current during first half-oscillation = 430 ampères.
Maximum magnetising force " " = 2070 C.G.S. units.

Large condensers, Nos. I–VI.—$T_0 = 3{\cdot}84 \times 10^{-5}$ sec., volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}10$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·062	0·124	$6{\cdot}1 \times 10^{-5}$	11·2	7·0
0·126	0·141	$6{\cdot}9 \times 10^{-5}$	14·6	10·2
0·168	0·191	$9{\cdot}4 \times 10^{-5}$	28·0	21·0
0·264	($\frac{1}{4}$)0·15	$14{\cdot}6 \times 10^{-5}$	67·0	56·0

Maximum current during first half-oscillation = 600 ampères.
Maximum magnetising force " " = 2890 C.G.S. units.

Large condensers, Nos. I–VIII.—$T_0 = 4{\cdot}46 \times 10^{-5}$ sec., volts = 7800, $L = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}06$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.	H.
0·06	0·129	$6{\cdot}2 \times 10^{-5}$	8·5	4·9	3700
0·146	0·163	$7{\cdot}9 \times 10^{-5}$	14·0	9·5	2180
0·178	0·204	$9{\cdot}8 \times 10^{-5}$	22·0	17·0	1100
0·24	0·319	$15{\cdot}3 \times 10^{-5}$	54·0	48·0	450
0·44	($\frac{1}{4}$)0·251	$24{\cdot}2 \times 10^{-5}$	137·0	120·0	—

Maximum current during first half-oscillation = 775 ampères.
Maximum magnetising force " " = 3700 C.G.S. units.

A similar series of results was obtained with a 1·5 mm. spark gap.

Large condenser, No. I.—$T_0 = 1{\cdot}58 \times 10^{-5}$ sec., volts = 6150, $L = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}36$.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·048	0·097	$5{\cdot}36 \times 10^{-5}$	53	43
0·098	($\frac{1}{4}$)0·049	$5{\cdot}4 \times 10^{-5}$	54	44

Maximum current during first half-oscillation = 92 ampères.
Maximum magnetising force " " = 440 C.G.S. units.

Large condenser, Nos. I–II.—$T_0 = 2{\cdot}28 \times 10^{-5}$ sec., volts = 6150, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}38$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·06	0·122	$6{\cdot}8 \times 10^{-5}$	54·1	33
0·126	($\frac{1}{4}$)0·064	$7{\cdot}1 \times 10^{-5}$	45·0	36

Maximum current during first half-oscillation = 140 ampères.
Maximum magnetising force " " = 670 C.G.S. units.

Large condensers, Nos. I–IV.—$T_0 = 3{\cdot}18 \times 10^{-5}$ sec., volts = 6150, $L = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}31$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·048	0·117	$6{\cdot}3 \times 10^{-5}$	18	13
0·158	0·19	$10{\cdot}3 \times 10^{-5}$	49	40
0·21	($\frac{1}{4}$)0·11	$11{\cdot}9 \times 10^{-5}$	65	54

Maximum current during first half-oscillation = 330 ampères.
Maximum magnetising force " " = 1580 C.G.S. units.

Large condenser, Nos. I–VI.—$T_0 = 3{\cdot}84 \times 10^{-5}$ sec., volts = 6150, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}52$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·06	0·12	$7{\cdot}1 \times 10^{-5}$	15·4	10·8
0·12	0·139	$8{\cdot}2 \times 10^{-5}$	21·0	15·6
0·176	0·207	$12{\cdot}2 \times 10^{-5}$	47·0	38·0
0·228	($\frac{1}{4}$)0·12			

Maximum current during first half-oscillation = 410 ampères.
Maximum magnetising force " " = 1980 C.G.S. units.

Large condenser, Nos. I–VIII.—$T_0 = 4{\cdot}46 \times 10^{-5}$ sec., volts = 6150, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}48$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·06	0·13	$7{\cdot}5 \times 10^{-5}$	12·6	8·5
0·150	0·17	$9{\cdot}8 \times 10^{-5}$	22·0	16·0
0·20	0·24	$14{\cdot}8 \times 10^{-5}$	51·0	41·0
0·29	0·36	$20{\cdot}8 \times 10^{-5}$	101·0	86·0
0·48	($\frac{1}{4}$)0·27 ?	$31{\cdot}0 \times 10^{-5}$	225·0	193·0

Maximum current during first half-oscillation = 500 ampères.
Maximum magnetising force " " = 2400 C.G.S. units.

Large condenser, Nos. I–X.—$T_0 = 5{\cdot}06 \times 10^{-5}$ sec., volts = 6150, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 2{\cdot}5$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·06	0·14	$8{\cdot}2 \times 10^{-5}$	11·7	7·5
0·18	0·19	$11{\cdot}1 \times 10^{-5}$	22·0	16·6
0·19	0·21	$12{\cdot}3 \times 10^{-5}$	27·0	21·0
0·23	0·26	$15{\cdot}2 \times 10^{-5}$	42·0	34·0
0·30	($\frac{1}{4}$)0·16	$18{\cdot}7 \times 10^{-5}$	63·0	52·0

Maximum current during first half-oscillation = 570 ampères.
Maximum magnetising force " " = 2750 C.G.S. units.

22. It is obvious from the above results that the important factor to be taken into consideration is the maximum current during the first half-oscillation; the points on the curve in fig. 4 plotted from these results agreeing fairly closely with those obtained from the first series of experiments. The variation in the depth to which the magnetism penetrates is relatively unimportant. It is to be noticed, however, that the amount of penetration depends on the permeability, and also on the frequency. Hence the magnetisation will penetrate into the iron and give a more nearly uniform distribution of magnetisation with the larger condenser.

23. The effect of using different diameters of wire for the cores may now be considered.

Below are given some of the results of a series of experiments made with a soft iron core consisting of 60 No. 18 wires. Area of iron = A_1 = 0·70 sq. cm. The same notation is adopted as in § 13.

Large condenser, No. I.—$T_0 = 1{\cdot}58 \times 10^{-5}$ sec., volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}81$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·04	0·08	$3{\cdot}4 \times 10^{-5}$	21	13·6
0·08	($\frac{1}{4}$)0·04	$3{\cdot}4 \times 10^{-5}$	21	13·6

Maximum current during first half-oscillation = 225 ampères.
Maximum magnetising force " " = 1080 C.G.S. units.

Large condenser, Nos. I–II.—$T_0 = 2{\cdot}28 \times 10^{-5}$ sec., volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}81$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·05	0·013	$4{\cdot}35 \times 10^{-5}$	16·6	10·5
0·011	($\frac{1}{4}$)0·057	$4{\cdot}8 \times 10^{-5}$	20·2	13·0

Maximum current during first half-oscillation = 245 ampères.
Maximum magnetising force " " = 1660 C.G.S. units.

Large condenser, Nos. I–X.—$T_0 = 4{\cdot}68 \times 10^{-5}$ sec., volts = 9400, $L = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}92$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·064	0·145	$6{\cdot}5 \times 10^{-5}$	8·5	4·4
0·178	0·209	$9{\cdot}4 \times 10^{-5}$	18·2	11·4
0·24	($\frac{1}{4}$)0·127	$11{\cdot}4 \times 10^{-5}$	26·0	17·2

Maximum current during first half-oscillation = 1070 ampères.
Maximum magnetising force " " = 5150 C.G.S. units.

24. These results appear, at first sight, somewhat remarkable, as it will be seen, on comparing them with the results obtained with a core composed of much finer wire, No. 28 S.W.G., that there is very little difference in the value of μ_e in the tables. On comparing the areas of the surface of the wires per centimetre length of the core, it is seen that, whereas for the No. 28 core the surface is 64·8 sq. cm., the surface for the No. 18 core is only 22·6 sq. cm.;

and since it is the surface of iron in the core which would appear to be most important, it might be expected that the change in frequency produced by the core would be much less noticeable.

The explanation, however, is obvious. With the larger diameter wire core the loss due to eddy currents is greatly increased. According to the approximate formula given above (§ 8) it depends on the fourth power of the diameter,* and hence the loss of energy during the first half-oscillation would be more than ten times as great for a core having the same amount of iron in it, and made of the No. 18 wire, as it would be for one of No. 28 S.W.G. wire. The actual magnetising current rapidly falls off, therefore, on account of this loss in energy.

There is another effect tending to diminish the magnetising force inside the iron, what is called the self-demagnetising force due to the magnetisation on the surface of the wire, at the place where the magnetic lines leak. This self-demagnetising force is increased with increase in diameter of the wire.

The diminution in the magnetising force owing to these causes will correspond with a greater permeability in the iron, which (although the increase in permeability of the iron diminishes the depth to which the magnetisation penetrates) is sufficient to counterbalance the much greater surface of iron exposed, with the finer core. As in this case the permeability must clearly be higher than in the previous cases, we may assume (taking into account the ratio of the surfaces), as a first approximation, $\mu = 30$.

[As the results obtained with the large condenser, No. I, seem to be in very fair accordance with those given for the other sparks, the result for the first half-oscillation has been used as a basis of calculation.]

Proceeding exactly as in § 16,

$$\mu = 30, \qquad p = 1{\cdot}84 \times 10^5, \qquad \sigma = 1{\cdot}2 \times 10^4.$$

Hence $$n^2 = 5800j, \qquad a = 0{\cdot}061 \text{ cm.}$$

The total induction through the core

$$= 8{\cdot}46\,\mathrm{H}\cos(pt - \phi), \qquad \tan\phi = 0{\cdot}85,$$

since the area of the iron $= 0{\cdot}70$ sq. cm. This gives a value for the effective permeability $\mu_e = 12{\cdot}1$, instead of $\mu_e = 13{\cdot}6$ as found by experiment.

It is of interest to observe the comparatively great lag in the maximum magnetic induction behind the maximum magnetising force. With these

* The radius of the finer wire No. 28 is 0·0193 cm., and of the No. 18 wire is 0·061 cm. The energy loss for the same mass of iron will be proportional to the squares of the diameters of the wires.

cores this lag is just over 40°. It should be remembered, however, in connection with this calculation that the permeability has been assumed constant, a condition very different from that which actually exists.

The actual loss of energy calculated from the approximate expressions given above gives a loss greater than the total energy stored in the condenser, which confirms the experimental observation that damping with these cores must be extremely rapid.

25. A third series of determinations has been made using cores made up of 250 No. 24 S.W.G. soft iron wires, in which it might be expected that the effects observed would be intermediate between those found with cores of larger and smaller diameter. The results have been arranged below in the same way as the previous ones.

The notation of § 13 has been adopted.

Large condenser, No. I.—$T_0 = 1{\cdot}58 \times 10^{-5}$ sec., volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}29$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·050	0·11	$3{\cdot}3 \times 10^{-5}$	20	15·4
0·12	($\frac{1}{4}$) (0·06)	$3{\cdot}6 \times 10^{-5}$	24	19·0

Maximum current during first half-oscillation = 190 ampères.
Maximum magnetising force ,, ,, = 910 C.G.S. units.

Large condenser, Nos. I–II.—$T_0 = 2{\cdot}28 \times 10^{-5}$ sec., volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$, $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}6$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·052	0·110	$4{\cdot}1 \times 10^{-5}$	14·6	10·6
0·132	0·064	$4{\cdot}8 \times 10^{-5}$	20·0	15·4

Maximum current during first half-oscillation = 305 ampères.
Maximum magnetising force ,, ,, = 1460 C.G.S. units.

Large condenser, Nos. I–IV.—$T_0 = 3{\cdot}18 \times 10^{-5}$ sec., volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}56$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·061	0·143	$5{\cdot}2 \times 10^{-5}$	12·0	8·0
0·184	0·19	$6{\cdot}9 \times 10^{-5}$	21·5	16·4
0·19	($\frac{1}{4}$)0·20	$7{\cdot}3 \times 10^{-5}$	24·1	19·0

Maximum current during first half-oscillation = 480 ampères.
Maximum magnetising force ,, ,, = 2300 C.G.S. units.

Large condenser, Nos. I–VI.—$T_0 = 3{\cdot}84 \times 10^{-5}$ sec., volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}60$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·08	0·173	$6{\cdot}5 \times 10^{-5}$	12·8	8·8
0·20	0·216	$8{\cdot}1 \times 10^{-5}$	20·0	15·4
0·22	0·228	$8{\cdot}5 \times 10^{-5}$	22·4	18·0
0·34	($\frac{1}{4}$)0·123	$9{\cdot}2 \times 10^{-5}$	26·0	21·0

Maximum current during first half-oscillation = 550 ampères.
Maximum magnetising force ,, ,, = 2650 C.G.S. units.

The value of A_1, the area of iron in this core, is equal to 0·61 sq. cm.

26. It will be seen that the values of the effective permeabilities hardly differ from those given in previous paragraphs for cores composed of finer and thicker wires; the permeability is slightly less than with the fine wire cores, and greater than with the thicker wire cores. It is obvious also that the absorption of energy due to eddy-current losses is intermediate between that observed with the thicker and thinner wires.

Taking $\mu = 25$, $a = 0{\cdot}028$ cm., and $H = 910$ C.G.S. units, the loss of energy during the first half-oscillation is $7{\cdot}5 \times 10^7$ ergs.

The total energy of the discharge in this case, however, is reduced to $3{\cdot}9 \times 10^7$, since the potential difference between the coatings of the condenser was only 7800 volts, and as in the case of the No. 18 core, the calculated eddy-current loss is too large, because allowance is not made for the penetration of the flux to the centre of the wire, and for the rapid dying away of the current.

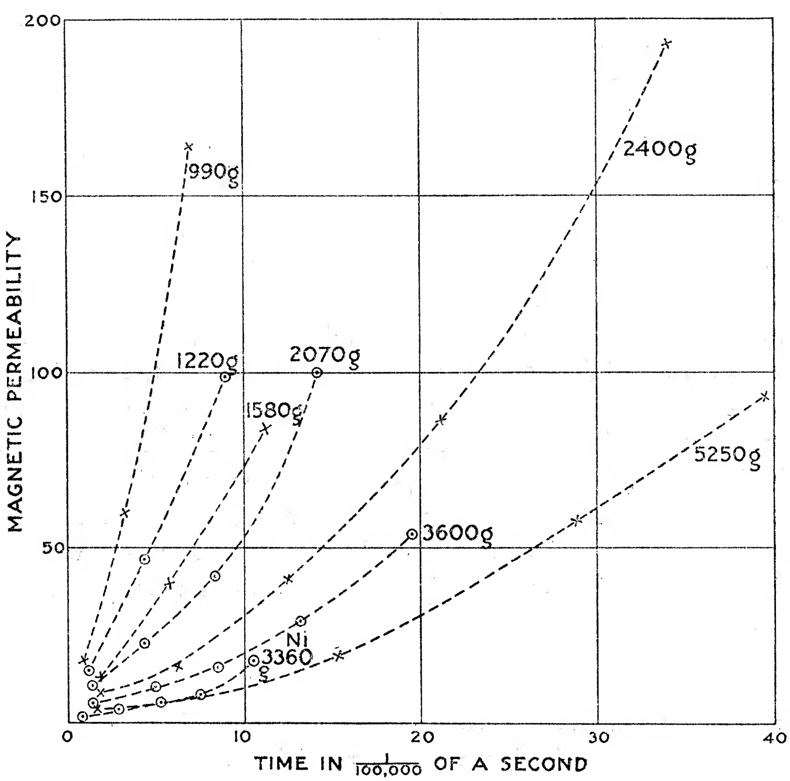

Fig. 5.—Curves showing increase in permeability during successive oscillations of discharge (from condenser) of iron wire (No. 28 B.W.G.) core placed in coil through which discharge passes.

Numbers opposite curves are maximum magnetising forces during first half-oscillation.

27. The results obtained for the three different sets of iron cores have all been plotted on the same sheet (fig. 4). The curves in fig. 5 have been drawn to show the way in which the effective permeability increases from the beginning of the discharge in one or two typical cases, the abscissæ representing time, and the ordinates "effective permeability." The maximum values are shown plotted against the time from the beginning of the discharge corresponding to the given maximum.

Effect of Size of Core.

28. An investigation was also made to determine, if possible, the depth to which the magnetisation penetrated into the core, *i.e.* to find out if the inner wires were screened by the outer layers; to this end, a series of cores were made up, consisting of wooden centres, round which were placed layers of iron wire.

With No. 28 wire, one set of cores was made up with a single layer placed round the inner core of mahogany, 85 wires being used.

A second set was made up of three layers of wires placed round a central wooden core, 220 wires being used.

A third core was made up of 24 No. 18 wires placed round a central core of wood.

It will be obvious that with a smaller volume of iron in the cores the accuracy of the calculation of the "effective permeability" of the iron will be very much reduced. For instance, the ratio of iron to air in the core with the 85 No. 28 wires is as 1 : 23 only, so that, in order to calculate the effective permeability of the core, the alteration in the self-induction of the coil must be determined with far greater accuracy than in the case in which the ratio of the iron to air is 1 : 3·5, as with the original cores.

With the No. 28 wire cores the general tendency of the results is to show that the induction through the outer layers is greater than through the central cores, with the No. 18 wires the effect was somewhat more marked, but the possible accuracy is so low that the numerical results are of little value.

Part III.—*Nickel Wire Cores.*

29. Besides the experiments made on iron as described in Part II, a series of discharges through coils having cores of nickel wire was photographed, the results from which are tabulated below. It will be seen that the phenomena are similar to those obtained with iron cores, though the effects are less marked with the nickel wires. The core consisted of 500 No. 28 wires, $A = 0{\cdot}58$ sq. cm.

Using the notation of (§ 13)—

Large condenser, No. I.—$T_0 = 1{\cdot}58 \times 10^{-5}$ sec., volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$ h. $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}31$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·036	0·077	$2{\cdot}35 \times 10^{-5}$	9·8	6·5
0·086	0·093	$2{\cdot}85 \times 10^{-5}$	14·7	11·1
0·096	0·097	$3{\cdot}0 \times 10^{-5}$	16·6	13·1
0·100	($\frac{1}{4}$)0·05			

Maximum current during first half-oscillation = 325 ampères.
Maximum magnetising force ,, ,, = 1560 C.G.S. units.

Large condenser, Nos. I–II.—$T_0 = 2{\cdot}28 \times 10^{-5}$ sec., volts = 9400, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}65$ sec.

l.	T.	$L \times 10^{-5}$.	μ_e.
0·08	$3{\cdot}1 \times 10^{-5}$	8·1	4·9
0·084	$3{\cdot}2 \times 10^{-5}$	8·7	5·3
0·088	$3{\cdot}4 \times 10^{-5}$	10·0	6·7
0·108	$4{\cdot}2 \times 10^{-5}$	15·4	12·0
0·14	$5{\cdot}4 \times 10^{-5}$	26·0	21·0

Maximum current during first half-oscillation = 485 ampères.
Maximum magnetising force ,, ,, = 2330 C.G.S. units.

Large condenser, Nos. I–VI.—$T_0 = 3{\cdot}84 \times 10^{-5}$, volts = 7800, $L_0 = 4{\cdot}1 \times 10^{-5}$ h., $l = 0{\cdot}6 \times 10^{-5}$ h., $t = 1{\cdot}60$ sec.

l.	l'.	T.	$L \times 10^{-5}$.	μ_e.
0·044	0·110	$4{\cdot}1 \times 10^{-5}$	4·7	1·7
0·124	0·128	$4{\cdot}8 \times 10^{-5}$	6·7	3·3
0·128	0·14	$5{\cdot}2 \times 10^{-5}$	8·0	4·9
0·14	0·14	$5{\cdot}2 \times 10^{-5}$	8·0	4·9
0·14	0·15	$5{\cdot}6 \times 10^{-5}$	9·4	6·1
0·18	0·24	$9{\cdot}0 \times 10^{-5}$	25·0	21·0
0·28	0·15	$11{\cdot}2 \times 10^{-5}$	39·0	34·0

Maximum current during first half-oscillation = 880 ampères.
Maximum magnetising force ,, ,, = 4200 C.G.S. units.

30. We may proceed in exactly the same way as in the case of the No. 28 iron wire cores to determine more exactly the actual permeability necessary to produce the observed results, making allowances for the "skin effect."

First, calculating the value of n,

$$\mu \text{ may be assumed } = 7 \text{ as a first approximation,}$$
$$p = 2{\cdot}67 \times 10^5,$$
$$\sigma = 1 \times 10^4 \text{ C.G.S. units,}$$

whence $$n^2 = 2350j, \qquad a = 0{\cdot}0193 \text{ cm.}$$

Hence the magnetic induction through a single wire $= 0{\cdot}0082 \cos(pt-\phi)$, where $\tan\phi = 0{\cdot}112$, giving an effective permeability which does not differ appreciably from 7. The lag in maximum magnetic induction behind maximum magnetising force is $6\frac{1}{2}°$ neglecting hysteresis.

Calculating next the eddy-current losses, we may again take the approximate expression given above.

The energy lost during the first half-oscillation

$$= 1{\cdot}29 \times 10^7 \text{ ergs},$$

the total energy of the discharge being $5{\cdot}93 \times 10^7$ ergs as before.

The loss of energy is about half that with iron wires of the same diameter, and the damping of the oscillations would therefore be expected to be much less rapid than with an iron wire core, and this result is shown experimentally by the spark photographs, the number of oscillations visible on the plate being approximately twice as large as with iron wire cores.

The results for nickel, therefore, may be said to have the same general characteristics as those obtained with iron, though the effects produced both on the frequency and the damping of the oscillations is less marked.

Cores of Other Materials.

31. Some of the early experiments made to verify the first observation of the variable frequency spark may now be considered.*

An experiment was made using a battery of 14 Leyden jars and a six-layer coil having a self-induction of rather less than 1 millihenry, in order to make certain that the effects observed with the iron cores were entirely due to the iron, and not to any other cause. For this purpose a core was made of insulated copper wire (No. 28 S.W.G.) and coated with shellac exactly as was done with the iron wire cores; this was inserted in the coil described above, but not the slightest change in frequency could be observed, and the number of oscillations visible on the plate was not greatly reduced.

Time for a complete oscillation without cores	$4{\cdot}01 \times 10^{-5}$ sec.
" " " with copper wire cores...	$3{\cdot}96 \times 10^{-5}$ "

The agreement between these results is clearly within limits of experimental error.

When a similar core of iron wires was inserted, all the characteristic effects both of change of frequency and of damping were observed.

When a core of solid brass was introduced, the time for a complete oscillation $= 3{\cdot}54 \times 10^{-5}$ seconds, thus showing an increase in frequency as found by Hemsalech.†

With a solid soft iron core the number of half-oscillations visible on the plate is reduced to two, thus showing the very great eddy-current loss; while

* Results published by Hemsalech confirm some of these observations. 'Journ. d. Phys.,' 1908, pp. 76–90; 'Comptes Rendus,' vol. 140, pp. 1322–1325.

† 'Comptes Rendus,' vol. 140, pp. 1322–1325.

the time for a complete oscillation $= 4{\cdot}4 \times 10^{-5}$ seconds. This appears to indicate a slight decrease in frequency, due to the permeability of the iron.

With the solid iron core, the permeability, therefore, is sufficiently great to overcome the effect of eddy currents, and tends to increase the self-induction, and to produce a slight decrease in the frequency.

32. It may be of interest here to notice some experiments in which a coil similar to the above was wound on a brass tube, about 1/32 inch thick. In this case the brass seemed to screen off the action of the iron entirely, and no difference whatever could be observed when the iron cores were introduced, either in damping or in change of frequency, this result is also in agreement with that obtained by Hemsalech using a tube of zinc (*loc. cit.*).

Conclusions.

General.

1. With an air condenser, the capacity of which was measured ballistically, and an air-core self-induction coil of known value, the frequency of the oscillations of the condenser discharge agrees (within limits of experimental error) with the values calculated by Kelvin's formula.

2. The resistance of a spark between spheres of 1 inch diameter, 2·5 mm. apart, has been estimated; the resistance of such a spark when the maximum current is greater than 200 ampères does not exceed 2 ohms. If the maximum current through the spark is greater than 500 ampères, the resistance does not exceed 0·75 ohm.

Effects Produced by an Iron Wire Core in the Self-induction.

3. When an iron wire is inserted in the self-induction coil, the time for each consecutive half-oscillation increases with the duration of the discharge.

4. With a series of discharges from a given condenser in which the maximum value of the magnetising current varies, the time for the first half-oscillation *decreases* with *increase* in the strength of the current.

5. The increase in the time of a half-oscillation with the duration of the discharge is due to the increase in the permeability of the iron, as the current, and consequently the magnetising force, dies away.

6. The permeability of the iron wires calculated from the observed increase in the self-induction of the coil, decreases with increase of the magnetising force. The curve connecting μ the "effective" permeability, with H, the magnetising force, agrees, within limits of experimental error, with that obtained on the assumption that the permeability of the iron is the same as it is under a steady magnetising force.

7. When an iron wire core is inserted into the self-induction coil the discharge is more quickly damped than with no core. This is due to eddy-current and hysteresis losses, the former being the more potent.

Effect of Variation of the Diameter of the Wire of the Core.

8. The "effective" permeability of thick wires, 0·12 cm. in diameter, is nearly equal to that of thin wires subjected to the same magnetising force (*i.e.* between the limits of 500–5000 gauss).

This is due to a balancing of the three effects stated below :—

(1) The increase in the "skin" effect with thick wire diminishes the apparent permeability.

(2) The maximum value of the magnetising force is reduced, in consequence of the increased dissipation of energy with thick wires, thus increasing the apparent permeability.

(3) The self-demagnetising force due to leakage from the surface of the wire decreases the actual magnetising force acting on the iron, thus increasing the permeability of the iron.

9. With cores of solid iron inserted into the self-induction coil the duration of the discharge is less than with any other cores ; the self-induction of the coil with solid iron cores is generally slightly (about 15 per cent.) greater than with air cores.

Effects Produced by Cores of other Materials.

10. Nickel wire cores produce similar effects to those observed with iron, but of diminished intensity, the permeability of the nickel wire with rapidly oscillating magnetic force being similar to that found with steady magnetising forces.

11. With cores made up of insulated copper wires, no changes, either in damping or self-induction, are apparent.

In conclusion, the author's best thanks are due to the late Lord Blythswood for the loan of the apparatus described above, and to the late Lord Kelvin for the kind encouragement given by him during the early stages of the research, to Mr. Duddell for the measurements of the self-induction of the various coils used, and for his criticisms, and to Dr. Silvanus Thompson for his criticisms and suggestions.